Complex Intuitionistic Fuzzy Set Theory for Multi-Criteria Decision Making Under Uncertainty

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ABSTRACT

As a more all-encompassing framework for capturing ambiguous and uncertain information, Complex Intuitionistic Fuzzy Set Theory (CIFST) emerged from the merging of intuitionistic fuzzy sets with complex fuzzy sets. Complex Intuitionistic Fuzzy Sets are defined and theorised in this study, along with some basic theorems about boundedness and the non-negativity of hesitation degrees. In addition, the paper delves into the possible uses of CIFS in decision-making scenarios including imprecise information and various criteria. Future research in intelligent systems, pattern recognition, and decision support approaches may be built upon the solid mathematical basis provided by the suggested framework, which also offers improved flexibility for modelling real-world situations.

Keywords: *Complex Intuitionistic, Decision, Hesitation, Uncertainty, Risk.*

I. INTRODUCTION

Both Lotfi A. Zadeh's groundbreaking ideas of Fuzzy Set Theory (FST) and Fuzzy Logic Theory (FLT) constitute a huge step forward in the area of mathematical modelling and AI. Elements in fuzzy logic can have membership degrees that go all the way from 0 to 1, as opposed to classical logic's use of true or false as binary truth values. Fuzzy logic is able to represent the inherent ambiguity and uncertainty in many real-life situations because of this feature. Therefore, FLT is an extension of discrete multi-valued logic systems and classical Boolean logic. When it comes to modelling complicated systems without clear-cut category borders, fuzzy logic's partial membership capability makes it a more practical and adaptable framework.

Researchers have greatly broadened fuzzy set theory's theoretical and practical reach throughout the years. To make fuzzy models better at representing complicated kinds of imprecision and uncertainty, several improvements have been suggested. The theory of intuitionistic fuzzy sets, as forward by Krassimir T. Atanassov, is among the most consequential expansions. Intuitionistic fuzzy sets differ from classical fuzzy sets in that they take into account both the membership and non-membership degrees for each element. Any hesitancy or doubt about the information is shown in the disparity between these two numbers. Not only have intuitionistic fuzzy sets come out, but fuzzy mathematics as a whole has seen a number of other significant advancements. Type-2 fuzzy sets, Z-numbers, complex fuzzy numbers, and linguistic variables are all examples of such. These enhancements all let fuzzy logic systems describe more complex uncertainty and ambiguity, which is a huge boon to their modelling capabilities.

Fuzzy set theory and fuzzy logic's mathematical underpinnings have also been considerably fortified, coinciding with these theoretical advancements. Theoretical frameworks and strict axiomatic systems have been developed by researchers in an effort to formalise these theories. While expanding their capacity to handle uncertain and imprecise input, fuzzy systems are formalised to assure compliance with recognised concepts of classical mathematics. Control systems, pattern recognition, AI, medical diagnostics, and decision support systems are just a few of the many real-world domains where fuzzy set theory and fuzzy logic have found use. The practical value of fuzzy logic approaches has been demonstrated by the maturation and successful implementation of several of these applications in commercial technology.

Because of its superior performance when dealing with unclear or unreliable data, Intuitionistic Fuzzy Set Theory (IFST) has become one of the most popular extensions of fuzzy set theory. In many real-life scenarios, the data utilised to ascertain the level of membership in a fuzzy set could be lacking, unclear, or impacted by noise. Due to their exclusive focus on membership values, traditional fuzzy sets could miss this kind of ambiguity. Fuzzy sets that rely on intuition overcome this shortcoming by taking into account the degree of membership and non-membership, as well as the uncertainty margin caused by missing data, all at once.

II. PRELIMINARIES

Let X be a non–empty universe of discourse. A Complex Intuitionistic Fuzzy Set (CIFS) A in X is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

Where

$$\mu_A(x) = r_\mu(x)e^{i\theta_\mu(x)}$$

$$\nu_A(x) = r_\nu(x)e^{i\theta_\nu(x)}$$

represent the complex-valued membership and complex-valued non-membership functions, respectively, such that

$$0 \leq r_\mu(x) + r_\nu(x) \leq 1$$

for every $x \in X$.

The hesitation degree is given by

$$\pi_A(x) = 1 - r_\mu(x) - r_\nu(x)$$

III. LEMMAS AND MATHEMATICAL PROPERTIES

Lemma 1: Boundedness Property

For every element $x \in X$ in a Complex Intuitionistic Fuzzy Set A, the amplitudes of membership and non-membership functions satisfy

$$0 \leq r_\mu(x) \leq 1$$

$$0 \leq r_\nu(x) \leq 1$$

Proof

By definition of CIFS, both membership and non-membership functions lie within the unit circle in the complex plane. Therefore, their amplitudes must satisfy

$$0 \leq r_\mu(x) \leq 1$$

and

$$0 \leq r_\nu(x) \leq 1$$

Additionally, the intuitionistic constraint requires

$$r_\mu(x) + r_\nu(x) \leq 1$$

Hence both amplitudes remain bounded within the interval [0,1].

Thus the lemma is proved.

Lemma 2: Non-negativity of Hesitation Degree

For any Complex Intuitionistic Fuzzy Set

$\pi A(x) \geq 0$

Proof

The hesitation degree is defined as

$\pi_A(x) = 1 - r_\mu(x) - r_\nu(x)$

From the intuitionistic constraint

$r_\mu(x) + r_\nu(x) \leq 1$

Therefore

$1 - r_\mu(x) - r_\nu(x) \geq 0$

Hence

$\pi_A(x) \geq 0$

Thus, the hesitation degree is always non-negative.

IV. COMPLEX INTUITIONISTIC FUZZY SETS IN DECISION MAKING

Uncertain, imprecise, or partial data is common while making decisions in contemporary systems. It is possible that traditional mathematical models misrepresent reality because they depend on absolute quantities and deterministic assumptions. There are a lot of decision-making contexts where experts could be hesitant or vague when expressing their opinions, and when the data that is given might have periodic qualities and uncertainty. A robust paradigm for dealing with such problems is Complex Intuitionistic Fuzzy Sets (CIFS), which include complex-valued representations that record amplitude and phase information in addition to membership, non-membership, and hesitation degrees. Compared to conventional fuzzy and intuitionistic fuzzy methods, decision models are able to portray uncertainty more thoroughly because to this enhanced representation.

Role of CIFS in Multi-Criteria Decision Making

One typical area where CIFS may be implemented efficiently is Multi-Criteria Decision Making (MCDM). Decision makers often face situations where they need to weigh many options using multiple metrics, including quality, performance, cost, risk, and sustainability. In many cases, these standards rely on subjective assessments and partial facts. By using CIFS, decision-makers are able to indicate how strongly they favour or disagree with each choice based on each criterion. Furthermore, when the expert lacks the clarity to make an accurate evaluation, the degree of hesitancy indicates their level of doubt. Decision information that is dynamic or time-dependent may also be modelled thanks to the complex representation.

Each option is assessed using a set of criteria utilizing intricate intuitionistic fuzzy values in a decision-making process that is based on CIFS. These numbers represent the extent to which a potential solution meets or falls short of a requirement. The results from several experts and criteria can be combined using aggregation operators. Score functions, similarity measurements, and distance measures from the alternatives to an ideal solution can be used to determine their ultimate ranking.

CIFS-Based Decision Making Model

Several methodical processes make up a typical CIFS-based decision-making framework. The decision problem is initially specified by establishing the collection of options and the criteria for evaluation. The following step is to compile expert opinions and express them using complicated intuitionistic fuzzy values. For each criterion, these numbers indicate the degree to which each alternative is a member or not.

A CIFS decision matrix must be built following the collection of assessment data. For each criterion, this matrix includes the intricate intuitionistic fuzzy assessments of all options. If the decision-making process involves numerous experts, the assessments can be combined. Criteria can also be given weighting factors to show how important they are.

A ranking technique is implemented when the aggregated decision matrix is obtained. To find out how much you favour each option, you may use a variety of tools, such scoring functions, accuracy functions, or distance measurements. The best choice is the one that has the highest preference score.

Application in Medical Diagnosis

One of the most important uses of CIFS in decision making is in systems that help doctors figure out what's wrong with a patient. Making medical decisions is frequently hard since you don't have all the information about the patient, the symptoms are different, and the experts don't always agree. CIFS can accurately depict this ambiguity by enabling clinicians to convey their confidence and doubt over the existence or non-existence of certain illnesses.

For example, when figuring out what sickness someone has based on a number of symptoms, each symptom can be represented by complicated intuitionistic fuzzy values that show how closely the symptom is related to the disease. The degree of doubt shows how unsure you are when the symptom doesn't clearly point to a certain ailment. A decision support system can use CIFS-based aggregation algorithms to figure out how likely different diseases are and help doctors choose the most likely diagnosis.

Application in Pattern Recognition

Pattern recognition is another significant area where CIFS-based decision models are quite useful. In a lot of identification systems, such facial recognition, speech recognition, or picture classification, the data that is provided may include noise, be unclear, or have patterns that overlap. CIFS can show both positive and negative evidence for pattern membership, as well as ambiguity.

A CIFS-based pattern recognition system uses a complicated intuitionistic fuzzy vector to show each pattern. When a new observation is added, the distance or similarity between it and known pattern classes is measured. The recognised pattern is the one that is most similar to or least far apart from the other patterns. This method makes it easier to recognise things in situations where the data is unclear or only half correct.

Application in Risk Assessment and Management

Risk assessment is a crucial aspect of decision making in fields such as finance, engineering, and project management. Risk evaluation often involves uncertain estimates of probabilities, impacts, and future conditions. CIFS provides a flexible approach for representing these uncertainties by incorporating both positive and negative assessments along with hesitation.

In risk management systems, different risk factors can be evaluated using CIFS values that represent the likelihood of occurrence and the severity of potential consequences. Decision makers can aggregate these evaluations to determine the overall risk level of different alternatives or projects. The CIFS framework allows organizations to make more informed decisions by capturing the uncertainty inherent in risk evaluations.

Application in Engineering and Control Systems

Complex intuitionistic fuzzy decision models are also widely applicable in engineering design and control systems. Many engineering problems involve multiple conflicting objectives and uncertain environmental conditions. CIFS-based decision frameworks can help engineers evaluate design alternatives under uncertain conditions and select the most suitable solution.

For example, in control system design, CIFS can be used to represent uncertain system parameters and evaluate different control strategies. The complex-valued representation is particularly useful when dealing with signals or processes that exhibit periodic or oscillatory behavior. By integrating CIFS into control algorithms, engineers can design more robust and adaptive systems.

Advantages of Using CIFS in Decision Making

The use of Complex Intuitionistic Fuzzy Sets in decision making offers several advantages over traditional fuzzy approaches. First, CIFS incorporates both membership and non-membership degrees, providing a richer representation of uncertainty. Second, the hesitation degree allows decision makers to express incomplete knowledge or indecision, which is common in real-world situations. Third, the complex representation captures additional information such as phase and periodic characteristics that cannot be represented in conventional fuzzy systems.

These features make CIFS particularly suitable for solving complex decision problems that involve uncertain, ambiguous, and dynamic information. As a result, CIFS-based decision models have gained increasing attention in areas such as artificial intelligence, management science, medical decision support, and engineering optimization.

V. CONCLUSION

Complex Intuitionistic Fuzzy Set Theory represents a significant advancement in the field of fuzzy mathematics by combining the strengths of complex fuzzy sets and intuitionistic fuzzy sets into a unified framework. This hybrid approach enhances the ability to model uncertainty, ambiguity, and incomplete information more effectively than conventional fuzzy models. By incorporating complex-valued membership and non-membership functions, CIFS allows the representation of both amplitude and phase information while simultaneously capturing the hesitation degree associated with uncertain data. The integration of Complex Intuitionistic Fuzzy Sets with artificial intelligence and machine learning techniques may further enhance their applicability in intelligent decision-support systems. Consequently, CIFS is expected to play an increasingly important role in addressing complex real-world problems involving uncertainty and incomplete knowledge.

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